20/06

$y' + \alpha(x) y = \beta(x)$

$$y(x) = e^{-\int \alpha(x)dx} \left\{ c + \int \beta(x) e^{\int \alpha(x) dx} dx \right\}$$

USA QUESTE FORMULE

PRIM' ORDINE ↑

SECOND' ORDINE ↓

$a_0 y''(t) + a_1 y'(t) + a_2 y(t) = f(t)$ $\quad a_0, a_1, a_2$ coeff. costanti

Eq. differenz. omogenea associata: $a_0 z''(t) + a_1 z'(t) + a_2 z(t) = 0$

$$a_0 \lambda^2 + a_1 \lambda + a_2 = 0$$

$\lambda_1 \neq \lambda_2$ reali distinti $\Rightarrow z(t) = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}$

$\lambda_1 = \lambda_2$ reali coincidenti $\Rightarrow z(t) = e^{\lambda t}(c_1 + c_2 t)$

λ_1 e λ_2 non reali $\Rightarrow \lambda = \alpha \pm i\beta \Rightarrow z(t) = e^{\alpha t}\left(c_1 \cos(\beta t) + c_2 \sin(\beta t)\right)$

- $y'' - 3y' + 2y = 0$

$\hookrightarrow \lambda^2 - 3\lambda + 2 = 0 \quad \Rightarrow \lambda = \begin{cases} 2 \\ 1 \end{cases}$

$y(t) = c_1 e^{2t} + c_2 e^{t}$

mettiamo che Cauchy ha il problema $\begin{cases} y(0) = 1 \\ y'(0) = 0 \end{cases}$ $\quad y'(t) = 2c_1 e^{2t} + c_2 e^{t}$

Allora $\begin{cases} 1 = c_1 + c_2 \\ 0 = 2c_1 + c_2 \end{cases} \Rightarrow \begin{cases} c_1 = -1 \\ c_2 = 2 \end{cases}$

$$y(t) = -e^{2t} + 2e^{t}$$

- $y'' - 4y' + 5y = 0$

$\hookrightarrow \lambda^2 - 4\lambda + 5 = 0$

$\lambda = 2 \pm i$ $\quad y(t) = e^{2t}\{c_1 \cos t + c_2 \sin t\}$

• $y'' - ky = 0$ $\quad$ k reale

$\lambda^2 - k = 0$

$\lambda = \pm\sqrt{k}$

- $k=0 \Rightarrow$ 2 reali coincidenti
- $k>0 \Rightarrow$ " " distinte
- $k<0 \Rightarrow$ 2 complesse

$k=0 \qquad y(t) = c_1 + c_2 t$ $\quad$ FASCIO DI RETTE

$k>0 \qquad y(t) = c_1 e^{\sqrt{k}t} + c_2 e^{-\sqrt{k}t}$

$k<0 \qquad y(t) = c_1 \cos\sqrt{-k}\,t + c_2 \,\text{sen}\sqrt{-k}\,t$

ho portato fuori le i ⚠

• $\begin{cases} y'' - 2y' + 4y = 0 \\ y(0) = 1 \\ y'(0) = 4 \end{cases} \Rightarrow \lambda^2 - 2\lambda + 4 = 0$

$\lambda = 1 \pm \sqrt{3}\, i$

$y(t) = e^t \{ c_1 \cos(\sqrt{3}t) + c_2 \sin(\sqrt{3}t) \}$

$y'(t) = e^t \{ c_1 \cos(\sqrt{3}t) + c_2 \sin(\sqrt{3}t) \} + e^t\{ -\sqrt{3} c_1 \sin(\sqrt{3}t) + \sqrt{3} c_1 \cos(\sqrt{3}t) \}$

$y(0) = 1 \Rightarrow \begin{cases} 1 = c_1 \\ 4 = c_1 + \sqrt{3} c_2 \end{cases} \qquad c_1 = 1$
$y'(0) = 4 \Rightarrow \qquad\qquad\qquad\qquad c_2 = \sqrt{3}$

l'integrale particolare è l'int. generale con però c_1 e c_2 = a qualcosa

• $y'' - 2y' + y = e^t$ $\quad$ eq. completa

$\begin{cases} z'' - 2z' + z = 0 \Rightarrow z(t) = e^t(c_1 + c_2 t) \\ \lambda^2 - 2\lambda + 1 = 0 \Rightarrow \lambda = 1 \text{ radice doppia} \end{cases}$

$y(t) = z(t) + u(t)$ $\quad$ ora calcolo $u(t)$ integr. particolare

$u(t) = At^2 e^{1t}$

$u'(t) = Ae^t(t^2 + 2t)$

$u''(t) = Ae^t(t^2 + 2t + 2t + 2)$

Sostituisco u nella formula

$Ae^t(t^2 + 4t + 2) - 2Ae^t(t^2 + 2t) + At^2 e^t = e^t$

$\Rightarrow$ calcolando $2A = 1 \Rightarrow A = \frac{1}{2} \Rightarrow u(t) = \frac{1}{2} t^2 e^t$

L'int. generale sarà allora $y(t) = e^{t}(c_1 + c_2 t) + \frac{1}{2} t^2 e^{t}$

$$\begin{cases} y'' + 3y' + 2y = 2\sin 4t \\ y(0) = 0 \\ y'(0) = 0 \end{cases}$$

omog. associata $z'' + 3z' + 2z = 0$

$\lambda^2 + 3\lambda + 2 = 0$

$\lambda = \frac{-3 \pm 1}{2} < \begin{matrix} -2 \\ -1 \end{matrix} \Rightarrow z(t) = c_1 e^{-2t} + c_2 e^{-t}$

Adesso troviamo il particolare $u(t) = A\sin 4t + B\cos 4t$

$u'(t) = 4A\cos 4t + 4B\operatorname{sen} 4t$

$u''(t) = -16A\sin 4t - 16B\cos 4t$

Quindi nell' equazione

$\Downarrow$ $-16A\sin 4t - 16B\cos 4t + 12A\cos 4t - 12B\sin 4t + 2A\sin 4t + 2B\cos 4t = 2\sin 4t$

onde per cui $\begin{cases} -14A - 12B = 2 \\ -14B + 12A = 0 \end{cases}$ $\begin{cases} B = 6/7\, A = -6/85 \\ A = -7/85 \end{cases}$

$u(t)$ sarà $u(t) = -\frac{1}{85}(7\sin(4t) + 6\cos(4t))$

Quindi $\underbrace{y(t) = c_1 e^{-2t} + c_2 e^{-t} - \frac{1}{85}(7\sin 4t + 6\cos 4t)}$

Ora risolviamo il problema di Cauchy

$\begin{cases} y(0) = 0 \\ c_1 + c_2 - \frac{6}{85} = 0 \end{cases}$ $\begin{cases} y'(0) = 0 \\ -2c_1 - c_2 - \frac{28}{85} = 0 \end{cases}$ $\Rightarrow \begin{cases} c_1 + c_2 = \frac{6}{85} \\ 2c_1 + c_2 = -\frac{28}{85} \end{cases}$ $\Rightarrow \begin{cases} c_1 = -\frac{2}{5} \\ c_2 = \frac{8}{7} \end{cases}$

dalla $y'(t)$ [...]

- $y'' - y = \cos t$

$z'' - z = 0$

$\lambda^2 - 1 = 0$

$\lambda = \pm 1$

$z(t) = c_1 e^{-t} + c_2 e^{t}$

$u(t) = A\cos t + B\sin t$

$u'(t) = -A\sin t + B\cos t$

$u''(t) = -A\cos t - B\sin t$ cioè $(-u(t))$

$y(t) = c_1 e^{-t} + c_2 e^{t} - \frac{1}{2}\cos t$ perché $-2u(t) = \cos t$

$-2A\cos t = \cos t \Rightarrow A = -\frac{1}{2}$

- $y'' + 2y' + 3y = 2t^2 - 1$

$\lambda^2 + 2\lambda + 3 = 0$

$\lambda = -1 \pm i\sqrt{2} \Rightarrow z(t) = e^{-t}\{c_1 \cos(\sqrt{2}t) + c_2 \sin(\sqrt{2}t)\}$

$u(t) = At^2 + Bt + C$

$u'(t) = 2At + B$

$u''(t) = 2A$

mettendo nell'equaz. $2A + 4At + 2B + 3At^2 + 3Bt + 3C = 2t^2 - 1$

da cui $\begin{cases} A = 2/3 \\ B = -8/9 \\ C = -\frac{5}{27} \end{cases} \Rightarrow y(t) = e^{-t}\{c_1 \cos(\sqrt{2}t) + c_2 \sin(\sqrt{2}t)\} + \frac{2}{3}t^2 - \frac{8}{9}t - \frac{5}{27}$