$\int_0 \int^{\sqrt{1-y^2}} \quad \int_1 dy + \int_0 x\,dx$

LE SERIE

30/05

$\sum_{i=0}^{\infty} a_n$

$\lim_{n \to +\infty} a_n = 0$ può convergere se $\lim_{n \to +\infty} a_n \neq 0 \to$ diverge

$\sum_{0}^{\infty} \frac{1}{n^3+3}$? $\quad \frac{1}{n^3+3} \sim \frac{1}{n^3}$ siccome $\sum_{1}^{\infty} \frac{1}{n^3}$ diverge anche

diverge

$\sum_{1}^{\infty} \frac{2n+4}{n+1} \qquad 2n+4 \sim 2n \qquad n+1 \sim n \qquad \frac{2n+4}{n+1} \sim 2$

$$\lim_{n \to +\infty} \frac{2n+4}{n+1} = 2$$

Trova il carattere della serie (cioè se conv. o diverge)

$\sum_{1}^{\infty} \frac{n!}{n^n}$? Fai $\lim_{n \to +\infty} \frac{a_{n+1}}{a_n} = \lim_{n \to +\infty} \frac{(n+1)!}{(n+1)^{n+1}} \cdot \frac{n^n}{n!} = \lim_{n \to +\infty} \frac{\cancel{(n+1)} \cdot n^n}{(n+1)^{n+1}} =$

$$= \lim_{n \to +\infty} \left(\frac{n}{n+1}\right)^n = \frac{1}{\lim_{n \to +\infty} \left(\frac{n+1}{n}\right)^n} = \frac{1}{e} < 1$$

$\Downarrow$

$\lim_{n \to +\infty} \frac{n!}{n^n} = 0$

CONVERGE

$\frac{\sqrt{n}}{\sqrt{n^2+n+1}} \qquad a_n \sim \frac{\sqrt{n}}{n} = \frac{1}{\sqrt{n}} \qquad \sum \frac{1}{n^{1/2}}$ diverge

$\sum_{n=1}^{\infty} \frac{1}{n} \left(\sin \frac{1}{n}\right)^2$ CONVERGE

$\frac{1}{n}\left(\sin \frac{1}{n}\right)^2 \sim \frac{1}{n} \cdot \frac{1}{n^2} = \frac{1}{n^3}$ C

tende a 0 se $n \to \infty$ $\quad \sin \frac{1}{n} \sim \frac{1}{n}$

$$\sum_{n=1}^{\infty} \frac{1}{n2^n + 2^{n^3}} \Rightarrow \text{converge pure lui}$$

CONVERGONO

$$\frac{1}{n2^n+2^{n^3}} \sim \frac{1}{n\cdot 2^n} \qquad n\cdot 2^n > 2^n \qquad \frac{1}{n2^n} < \frac{1}{2^n} \qquad \frac{1}{2^n} \text{ CONVERGE x DEFINIZ.}$$

—o—

$$\lim_{n\to+\infty} \frac{(n+2)^{n+1}}{(n+1)\cancel{!}} \cdot \frac{\cancel{n!}}{(n+1)^n} = \lim_{n\to+\infty} \frac{(n+2)^{n+1}}{(n+1)^{n+1}} = \lim_{n\to\infty} \left(\frac{n+2}{n+1}\right)^{n+1} = \lim_{n\to+\infty} \left(1+\frac{1}{n+1}\right)^{n+1} =$$

$$= e$$

—— o —— o —— o ——

$$\sum_{0}^{\infty} b_n \text{ — Se converge assolutamente} \Rightarrow \text{converge anke } \sum (-1)^n b_n$$

$$\sum_{1}^{\infty} (-1)^n \frac{1}{n} \qquad \lim_{n\to+\infty} \frac{1}{n} = 0 \qquad \left\{\frac{1}{n}\right\} \text{ diverge?} \qquad \frac{1}{n} > \frac{1}{n+1}$$

$$n+1 > n \quad \forall n$$

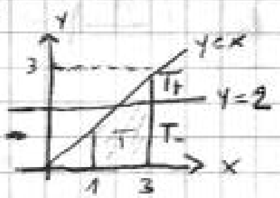
Converge!

$$\sum_{1}^{\infty} (-1)^n \frac{1}{n^2} \qquad \text{siccome } \frac{1}{n^2} \text{ converge assolutamente} \Rightarrow \text{CONVERGE}$$

$$\sum_{1}^{\infty} (-1)^n \frac{n+1}{2n} \qquad \text{non converge x il } \lim_{n\to+\infty} \text{ oscilla}$$

$$\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{n+1} \qquad \frac{\sqrt{n}}{n+1} \sim \frac{1}{\sqrt{n}} \qquad \text{converge}$$

Trova il volume del solido $z = \frac{y-2}{1+x^2}$



$$V(E) = \int_T |z|\,dT = \int_T \left|\frac{y-2}{1+x^2}\right| dT \qquad z \geq 0 \quad y-2 \geq 0 \quad y \geq 2$$

$$= \int_{T_+} \frac{y-2}{1+x^2}\, dT_+ + \int_{T_-} \frac{2-y}{1+x^2}\, dT_- \quad \text{— siccome } \int_{T_-} = \int_T - \int_{T_+}$$

$$= 2\int_{T_+} z\, dT_+ - \int_T z\, dT$$

$$\int_{T_+} = \int_2^3 dy \int_y^3 z\, dx$$

$$\int_{T_-} = \int_1^3 dx \int_0^1 z\, dy + \int_1^2 dy \int_y^3 z\, dx$$

$V(E) = \int_T |z| \, dT$

13/06

EQUAZ. DIFFER. LINEARI DEL I° ORDINE

$y' + a(x) y = b(x)$ oppure meglio $y' = \alpha(x) y + \beta(x)$

$$y(x) = e^{\int \alpha(x) dx} \cdot \left\{ c + \int e^{-\int \alpha(x) dx} \cdot \beta \, dx \right\}$$

Problema di Cauchy $y(x_0) = y_0$

- $y' = xy - 2x \qquad y(0) = 0$

$$y = e^{\int x dx} \left\{ c + \int -2x e^{-\int x dx} dx \right\} =$$

$$= y = e^{x^2/2} \left\{ c + 2 \int -x e^{-\frac{x^2}{2}} dx \right\} = e^{x^2/2} \left\{ c + 2 e^{-\frac{x^2}{2}} \right\}$$

$$= y = c e^{x^2/2} + 2$$

ossia sia $x=0$ $y=0$

Per trovare la c faccio $y(0) \Rightarrow 0 = c \cdot 1 + 2 \Rightarrow c = -2$

Trova eq. retta tg: $y - y_0 = m (x - x_0)$
$\uparrow$
$y'(x_0)$

$\Rightarrow y = 0 \quad (x_0 = 0, \; y_0 = 0)$

- $y' = 2y - x$

$$y(x) = e^{\int 2 dx} \left\{ c + \int -x e^{-2\int dx} dx \right\} = e^{2x} \left\{ c + \int -x e^{-2x} dx \right\} =$$

x parti

$$= e^{2x} \left(c + \frac{1}{2} x e^{-2x} + \frac{1}{4} e^{-2x} \right) = c e^{2x} + \frac{1}{2} x + \frac{1}{4}$$

- $y' = \frac{1}{x} y + 3$ $\log|x|$

$$y(x) = e^{\int \frac{1}{x} dx} \left\{ c + \int 3 e^{-\int \frac{1}{x} dx} dx \right\}$$

$$= |x| \left\{ c + 3 \int \frac{1}{|x|} dx \right\} \rightarrow \begin{cases} x > 0 & y(x) = x \{ c + 3 \log x \} \\ x < 0 & y(x) = -x \{ c - 3 \log(-x) \} \end{cases}$$

nel caso $x>0$ $\quad y(x) = \cancel{xx}\,x\,(c+3\log x) = x\cdot 3\log(Kx)$

- $y' = \frac{1}{x}y + 3 \quad (x\neq 0)$

Probl. Cauchy $\quad y(-1) = 1$

$$y(x) = 3x\log\{Kx\} \Leftarrow \quad y(x) = x\{c+3\lg|x|\}$$

$$\Downarrow$$

$$1 = -1\,(c+3\lg(1)) \Rightarrow \boxed{c=-1}$$

ossia $1 = -3\lg(-x) \Rightarrow K = -e^{-1/3}$

- $y' = \frac{xy}{1+x^2} \qquad y' = \alpha(x)\cdot y + \beta(x)$

$$y(x) = e^{\frac{1}{2}\int\frac{2x}{1+x^2}dx}\cdot\{c+\cancel{\int e^{-\int\ldots}}\cdot 0\,dx\}$$

$$y(x) = e^{\frac{1}{2}\log(1+x^2)}\,c \qquad \Rightarrow y(x) = c\sqrt{1+x^2}$$

- $y' = -\frac{2x}{1+x^2}y + \frac{1}{x(1+x^2)} \qquad$ Cauchy $y(-1) = 0 \qquad x\neq 0$

$$y(x) = e^{-\int\frac{2x}{1+x^2}dx}\left(c+\int\frac{1}{x(1+x^2)}e^{\int\frac{2x}{1+x^2}dx}dx\right)$$

$$y(x) = e^{-\lg(1+x^2)}\left(c+\int\frac{1}{x\cancel{(1+x^2)}}e^{\lg\cancel{(1+x^2)}}dx\right)$$

$$= \frac{1}{1+x^2}\left(c+\int\frac{1}{x}dx\right) \Rightarrow y(x) = \frac{1}{1+x^2}(c+\lg|x|)$$

$$0 = \frac{1}{2}c \Rightarrow \boxed{c=0}$$