

Esercizio 1

22 esame 4/3/03

a)  $\eta_x = 0$   $R_x(0) = 0 + 4 = 4 \rightarrow R_x[k] = 4\delta[k]$   
 $\sigma_x^2 = 4$   $R_x(\pm 1) = -1$   $\Rightarrow R_x[k] = -(\delta[k-1] + \delta[k+1])$   
 altri nulli

Però  $\bar{S}_x(f) = 4 - 2\cos(2\pi f)$

$\eta_y = \eta_x \lambda(0) = 0$   
 $\sigma_y^2 = \sigma_x^2 |k| \omega^2 = 4 \cdot \left| \frac{1}{1 - \frac{1}{2\omega^2}} \right|^2 = 4 \cdot \frac{4 \pm 2\sqrt{3}}{4} = 4 \pm 2\sqrt{3}$

b)  $H(f) = \frac{1}{1 + C(\cos(2\pi f) - j\sin(2\pi f))}$

$\bar{S}_y(f) = \bar{S}_x(f) |H(f)|^2 = \text{costante (x bianca)} \Rightarrow \bar{S}_y(f) = \frac{K}{|H(f)|^2} = 4 - 2\cos(2\pi f)$

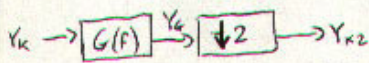
quindi  $[1 + C^2 + 2C\cos(2\pi f)]K = 4 - 2C\cos(2\pi f) \Rightarrow \begin{cases} (1+C^2)K = 4 \Rightarrow K = 2\pm\sqrt{3} \\ 2CK = -2 \Rightarrow C = -\frac{1}{K} = -\frac{1}{2\pm\sqrt{3}} \end{cases}$

MATLAB:

```
K = 10^6;
y = sqrt(2 + sqrt(3)) * randn(1, K);
media
var [...]
```

c) Passa basso Pa  $G(f) = \text{rect}\left(\frac{f}{2B}\right) = \text{rect}(2f)$

$\bar{S}_y(f) = \bar{S}_x(f) \cdot |G(f)|^2 = K \cdot \text{rect}(2f)$



poi bbo(?)

Soluzioni Esame 18 Luglio 2003 - Primo esercizio

ESER 1

$0 = \frac{dx-1}{x^2} \quad (x=1)$

da esame 18/07/03

a)  $R_x(0) = \sigma_x^2 + \eta^2 = 1$   
 $R_x(\pm 1)$  viene da  $\rho_{\pm 1} = \frac{R_x(\pm 1) - \eta^2}{\sigma_x^2} \Rightarrow R_x(\pm 1) = -0,5$   
 $R_x(j) = 0$  per  $j \neq \pm 1$  se  $R_x(j) = 1$  per  $j = \pm 1$

$\bar{S}_x(f) = R_x(0) + 2 \sum_{m=1}^{\infty} R_x(m) \cos(2\pi m f T)$   
 $\bar{S}_x(f) = 1 + 2 R_x(1) \cos(2\pi f T) + 2 \sum_{m=2}^{\infty} 0 = 1 - \cos(2\pi f T) + 4 \sum_k \delta(f - k)$

b)  $H(f) = 1 + C e^{-j2\pi f} = 1 + C \cos(2\pi f) - jC \sin(2\pi f)$

$|H(f)|^2 = 1 + C^2 \cos^2 + 2C \cos + C^2 \sin^2 = 1 + 2C \cos(2\pi f) + C^2$

$H(0) = 1 + C$

Essendo incoerente  $\Rightarrow \eta_x = \eta_A \cdot H(0) = 0 \Rightarrow (1+C)\eta_A = 0$

Poi con  $\eta_A = 0$  uso la condizione  $\sigma_x^2 = 1 = \sigma_A^2 \cdot |H(0)|^2 \Rightarrow 1 = (1+C)^2 \cdot \sigma_A^2$

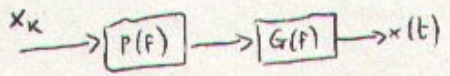
MATLAB:

```

k = 10^6;
x = randn(1, k);
media = mean(x);
var = std(x)^2;
sprintf('Media: %.f', media);
sprintf('Varianza: %.f', var);
    
```

c) il passa basso ha  $G(f) = \text{rect}(\frac{f}{2B}) = \text{rect}(\frac{f}{2})$   $B=1 \Rightarrow T=1$

il mantentitore ha  $P(f) = T \text{sinc}(fT) e^{-j\pi f T} = \text{sinc}(f) e^{-j\pi f}$   
 $|P(f)|^2 = \text{sinc}^2(f)$



$S_{x(t)}(f) = \bar{S}_{x_k}(f) |P(f)|^2 |G(f)|^2 = [1 - \cos(2\pi f)] \cdot \text{sinc}^2(f) \cdot \text{rect}(\frac{f}{2})$